京都大学
KYOTO UNIVERSITY

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Yukawa Institute for Theoretical Physics
Kyoto University, Kyoto 606-8502, Japan
email: tatsuma.nishioka@yukawa.kyoto-u.ac.jp

January 5, 2022

Dear Colleagues,

I am writing in strong support of Dr. Pawel Caputa's application for habilitation. His main research has been focused on the understanding of holographic duality between quantum field theory (QFT) and quantum gravity on a curved space from the viewpoint of quantum information theory. He has made pivotal contributions to revealing the fundamental structure of quantum entanglement in QFTs and developing new measures of quantum information in conformal field theories (CFTs). His habilitation thesis consists of ten excellent papers, and I would like to comment on the main points of his works in the following.

Entanglement entropy in QFTs has attracted broad attention in high energy theory and condensed matter physics as a novel type of "observable" which is sensitive to global correlation and hence distinguishes different quantum phases of matter. In contrary to static systems where there were a considerable number of works on entanglement entropy and its related measures in QFTs and holographic theories, the dynamical aspects of quantum information were less understood mostly due to its technical difficulties.

The five papers [A1 - A5] of the habilitation are concerned with the study of quantum entanglement measures such as entanglement entropy, Rényi entropy and a measure of information scrambling known as Out-of-Time-Ordered-Correlators (OTOC) under dynamical evolution triggered by changing a quantum state globally or locally at some time in CFTs.

In the first paper [A1], Dr. Caputa introduced a charged entanglement entropy to incorporate the dependence of a quantum state on a U(1) conserved charge in two-dimensional CFTs. He employed this new entanglement measure to probe thermalization process driven by a global quantum quench for a quantum state with conserved charges, and found a universal behavior, a linear growth at early time followed by a saturation. He also obtained a holographic counterpart of the charged entanglement entropy and confirmed that the holographic formula correctly reproduced the CFT results.

In a series of papers [A2-A4], Dr. Caputa initiated explorations of the dynamics of entanglement measures under local quantum quenches in two-dimensional CFTs. The main results of these papers derived from the holographic description of a locally excited operator as a massive particle propagating on the AdS spacetime, which he and his collaborators proposed for the first time. This novel holographic description reduced the calculation of entanglement entropy to that of a geodesic length on the AdS spacetime with the backreaction from a massive particle, resulting in a perfect match with the CFT analysis in the large central charge limit.

In the paper [A5], Dr. Caputa investigated two physical quantities, OTOC and the locally excited quantum entanglement as a measure of information scrambling in rational CFTs that are known to be integrable. Quantum chaos in quantum many-body systems has been actively studied in relation to the scrambling phenomenon, which represents the delocalization of quantum information in black holes. Two-dimensional CFTs holographically dual to black holes are expected to have a quantum chaotic property, and the possibility that this property can be read from the asymptotic behavior of OTOC has been discussed extensively. Dr. Caputa pointed out for the first time that in the limit of large central charge, where the theory is expected to have a holographic duality, the quantum entanglement shows the same scrambling phenomenon as in the dual gravity theory, although the OTOC does not show any chaotic behavior, reflecting the integrability of the theory. This result has an important implication that quantum chaos and scrambling are different concepts.

The results of these papers not only show the significance of quantum entanglement as a probe of dynamical evolution in QFT but can also be seen as nontrivial checks for the holographic formula of entanglement entropy as well as the AdS/CFT correspondence itself.

In the last five papers [A6-A10], Dr. Caputa examined the notion of complexity that quantifies how much quantum operation is required to construct a given quantum state in QFT, and proposed ingenious definitions in CFTs. While complexity is well-defined in discrete systems built out of quantum circuits in quantum mechanics, it is much less clear in continuous systems such as QFT, and there have been various proposals and conjectures in high energy theory community so far.

In the seminal papers [A6, A7], Dr. Caputa defined a complexity of a quantum state in QFTs by the path integral. The key idea of the path integral complexity is to optimize the background geometry QFTs live on by minimizing the action of complexity. In two-dimensional CFTs, the complexity action becomes the Liouville action, and the optimized metric is shown to be a hyperbolic space. Interestingly, this hyperbolic space can be seen as a slice of a three-dimensional AdS spacetime, which sheds some light on the mechanism behind the AdS/CFT correspondence. Furthermore, he generalized the Liouville action in two dimensions to higher dimensions and proposed a Liouville-type complexity action in higher-dimensional CFTs. The path integral complexity is the first definition of complexity that is applicable to CFTs including holographic ones and has led to subsequent developments. As a notable application of these works, in the paper [A8], Dr. Caputa applied the path integral optimization to the computation of entanglement of purification (EoP), and obtained a consistent result with its dual gravity result. Since EoP is an extremely hard quantity to calculate in QFTs, this result undoubtedly shows the virtue of the path integral optimization method.

In more recent papers [A9,A10], Dr. Caputa pursued the application and extension of Nielsen's geometric complexity that measures the distance between quantum states in QFTs. In the paper [A9] he analyzed the dynamics of quantum quenches in free Gaussian theory and found the complexity shows a universal scaling behavior and can be a useful probe of quench dynamics. Furthermore, Dr. Caputa combined the idea of geometric complexity with the path integral complexity and introduced Virasoro circuits as a series of conformal transformations in two-dimensional CFTs in the paper [A10]. The geometric complexity action in this setup is shown to be the Polyakov action of two-dimensional gravity, which suggests an extremely intriguing connection between quantum circuits and quantum gravity.

All the habilitation papers of Dr. Caputa include very strong results as well as groundbreaking proposals. They are published in high-quality journals in physics, and four of them have appeared in the most prestigious one, *Physical Review Letter*. I have no doubts that Dr. Caputa fully deserves the habilitation title and I therefore very strongly recommend it.

Sincerely,

Tatsuma Nishioka

Tatsuma Nishioka
Associate Professor
Ph.D